A Formal Framework for Data-Aware Process Interaction Models

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Abstract

IT support for distributed and collaborative workflows as well as related interactions between business partners are becoming increasingly important. For modeling such partner interactions as flow of message exchanges, different top-down approaches, covered under the term *interaction modeling*, are provided. Like for workflow models, correctness constitutes a fundamental challenge for interaction models; e.g., to ensure the boundedness and absence of deadlocks and lifelocks. Due to their distributed execution, in addition, interaction models should be *message-deterministic* and *realizable*, i.e., the same *conversation* (i.e. sequence of messages) should always lead to the same result, and it should be ensured that partners always have enough information about the messages they must or may send in a given context. So far, most existing approaches have addressed correctness of interaction models without explicitly considering the data exchanged through messages and used for routing decisions. However, data support is crucial for collaborative workflows and interaction models respectively. This technical report enriches interaction models with the data perspective. In particular, it defines the behavior of data-aware interaction models based on *Data-Aware Interaction Nets*, which use elements of both *Interaction Petri Nets* and *Workflow Nets with Data*. Finally, formal correctness criteria for Data-Aware Interaction Nets are derived, guaranteeing the boundedness and absence of deadlocks and lifelocks, and ensuring message-determinism as well as realizability.

Index Terms

Business Process Management, Distributed Workflows, Collaborative Workflows, Realizability, Soundness, Interaction Modeling, Data-Aware Process Models, Interaction Nets

I. Introduction

Workflow management is of utmost importance for companies that want to efficiently handle their workflows as well as their interactions with partners and customers [1]. Despite the varying issues relevant for the IT support of distributed and collaborative workflows [2], common aspects to be considered include the support of appropriate modeling techniques as well as the definition of a formal execution semantics, ensuring proper and correct partner interactions (i.e., message exchanges).

Workflow management methods and techniques tackling these challenges consider a *choreography* as a specification of message exchanges between the *partners* of a collaborative workflow. Respective approaches provide a global view on distributed workflows and support partners in correctly defining their private processes (*partner processes* for short). The latter can be transformed into distributed, executable workflows. When executing these workflows, their interplay is coordinated in terms of a *conversation* (i.e., a sequence of exchanged messages) that follows the global behavior specified by the choreography.

Currently, there exist two different paradigms for modeling choreographies: *interconnection modeling* and *interaction modeling*. *Interconnection modeling* uses message exchange as link between partner processes or public views on them. In particular, this paradigm does not allow modeling the message exchange separately from the partner processes. Hence, it is considered as a *bottom-up approach*. Approaches enabling interconnection modeling include BPMN Collaboration Diagram [3], BPEL4Chor [4], and Compositions of Open Nets [5]. By contrast, *interaction modeling* provides a *top-down approach*. An *Interaction Model* specifies the flow of message exchanges

without having any knowledge about the partner processes. Moreover, the models of the partner processes are created taking the interaction model into account. Nevertheless, common interaction models use the same patterns as workflow models (e.g. parallel and conditional branchings), but instead of tasks they refer to the messages exchanged. Approaches enabling interaction modeling include iBPMN [6], BPMN Choreography Diagrams [3], Service Interaction Patterns [7], and WSCDL [8].

This technical report focuses on the correctness of interaction models. Related issues discussed in the literature include boundedness and absence of deadlocks and lifelocks, as well as the *realizability* of interaction models [5], [9]–[11]. Realizability postulates that partners always can compute which messages they must or may send in a given execution context. Fig. 1 (1) outlines a simple example of a non-realizable choreography with four partners A, B, C, and D, and two messages m_1 and m_2 . This interaction model specifies that after sending message m_1 from A to B, message m_2 must be sent from C to D. Obviously, only A or B knows when C must send message m_2 , but C does not have this knowledge. Consequently, this interaction model is not realizable. A necessary precondition for realizability is *message-deterministic* behavior, i.e. the same conversation (i.e. sequence of messages) should always lead to the same result. An example of an interaction model, which is not message-deterministic, is shown in Fig. 1 (2); this interaction model comprises partners A, B, and C, and messages m_1, m_2, m_3 , and m_4 . After sending the first message m_1 , either the upper or the bottom branch shall be chosen. In any case, the next message m_2 must be sent from B to C. Depending on the branch chosen, however, then C either must send m_3 to B or m_4 to A. From the perspective of C, it cannot be determined, which of the two interpretations shall be applied. By contrast, B knows the chosen branch (e.g., the upper one). Hence, C might send m_4 to A, while B waits for m_3 , or vice versa.

A property similar to realizability is *clear termination*. It requires that a partner always can compute, whether he will be sender or receiver of any messages in the sequel. An example of an interaction model, which is not clearly terminating, is shown in Fig. 1 (3). This interaction model comprises partners A, B, and C, and messages m_1, m_2, m_3 , and m_4 . After sending the first message m_1 from A to B, B can either send message m_2 to A or message m_4 to C. When choosing the first option (i.e. B sends m_2 to A), A must send m_3 to C afterwards. In turn, when choosing the second option (i.e. B sends m_4 to C), the execution is terminated, although A may still wait for the arrival of message m_2 . Note, that from the perspective of A nothing has changed since m_1 was sent.

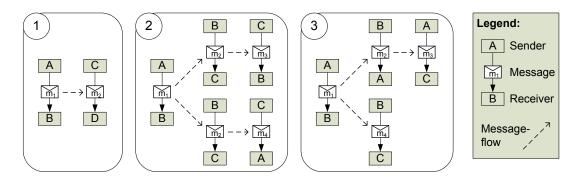


Fig. 1. Violating realizability, message-determinism, clear termination [9]

Existing approaches for interaction modeling do not adequately support the data perspective. Either related execution semantics completely ignore the data perspective or there is a lack of appropriate correctness criteria, especially if routing decisions are based on message data.

This technical report deals with fundamental correctness issues when making interaction models *data-aware*. Section II provides an example from the healthcare domain to emphasize the need of data-awareness in interaction models. Section II further discusses the challenges to be tackled when considering the data perspective. Section III then introduces our formal framework for *data-aware interaction modeling*. First, an interaction meta-model is provided in terms of the *Data-Aware Choreography* (DAChor). The behavior of a DAChor is described by a transformation to *Data-Aware Interaction Nets* (DAI Nets). These combine *Interaction Petri Nets* [9] and *Workflow*

Nets with Data [12]. Based on Data-Aware Interaction Nets, the set of allowed conversations (i.e., message exchanges) is derived and used to introduce formal correctness criteria for DAI Nets and DAChor respectively. These criteria guarantee for the boundedness and absence of deadlocks and lifelocks, and ensure message-determinism, realizability, and clear termination. Section IV discusses related work and Section V concludes with a summary and outlook.

II. EXAMPLE, CHALLENGES, CONTRIBUTION

This section introduces a simplified real-world scenario, which we elaborated in the context of case studies conducted in the healthcare domain. These case studies highlighted the relevance of the data perspective in interaction models. Thus, the scenario we select emphasizes the challenges arising from the support of data-awareness in interaction models. It describes the transport of a patient to and from a unit performing a Positron Emission Tomography (PET) scan. A PET scan is a kind of nuclear medicine imaging not performed by the respective hospital itself in our scenario. Thus, if a PET scan is ordered for a patient, patient transportation to the respective provider is required. In this context, the hospital must inform the provider of the PET scan about the patient's status, such that he can decide on the preparations required. Furthermore, we require a patient to be examined just before the transport to exclude potential risks (e.g., the patient being in a critical condition).

The scenario involves three partners, i.e., the Hospital responsible for the patient and ordering the PET scan, the Transportation (Transp.) Provider transporting the patient, and the PET provider performing the PET scan. The interaction starts with the Hospital requesting the PET scan (Request PET). In the context of this request, the Hospital informs the PET Provider about the status of the patient. In turn, the PET provider confirms the time for which the scan is scheduled (Confirm), and then requests the Transp. Provider to perform the transport (Request Trans.).

- If the patient is in a critical condition, the Transp. Provider requests the Hospital to examine him to check whether he is transportable (Request Exam.). Based on the Result of this examination, the Hospital informs the Transp. Provider on whether to continue or abort the interaction.
- If the interaction is continued or the patient is not in a critical condition, Transp. Provider informs the PET provider after picking up the patient and arriving at the PET unit (Arrival). After the PET scan is performed, the PET provider requests retransport of the Transp. Provider (Retransport). Finally, the Transp. Provider informs the Hospital about the return of the patient (Return).

Obviously, properly modeling the interactions of this scenario requires support for routing decisions based on the data of the messages exchanged. More precisely, in the given scenario, there is a decision referring to data of the first message exchanged (i.e. whether or not the patient is in a critical condition). Another decision refers to the message sent by the hospital and indicating whether the request shall be canceled. Hence, we use a notation based on BPMN 2.0 [3] and iBPMN [9], but enrich it with so-called *virtual data objects*. We denote this notation as *Data-Aware Choreography* (DAChor) and use it to model our scenario in Fig. 2. Virtual data objects have a data domain and can be used as variables when defining conditions for routing decisions. However, these virtual data objects are not used for modeling information flow. Thus, the *data assignment relation* denotes which data of an interaction is assigned to any virtual data object. Note that such a data assignment relation can only lead from an interaction to a virtual data object, but not vice versa. Furthermore, an *interaction* is assigned to a *message class* denoting the message type. From the message class, the sender, receivers, and data domain are inherited (e.g., boolean). Finally, when executing a choreography, messages of the related message class correspond to interactions.

Having a closer look at our scenario, one can recognize that it neither ensures realizability nor clear termination. If the Hospital requests canceling the PET scan, the PET provider is not informed accordingly and hence may still wait for the message; i.e., no clear termination is ensured. However, if *Alternative 2* (cf. Fig. 2) is applied, the PET provider will be informed and clear termination can be ensured. In turn, realizability is violated for the given interaction model, since Transp. Provider does not know whether the patient is in a critical condition. Thus, Transp. Provider cannot determine whether an examination must be requested. To ensure realizability, it is not sufficient to only check whether this information was directly sent to Transp. Provider. Consider

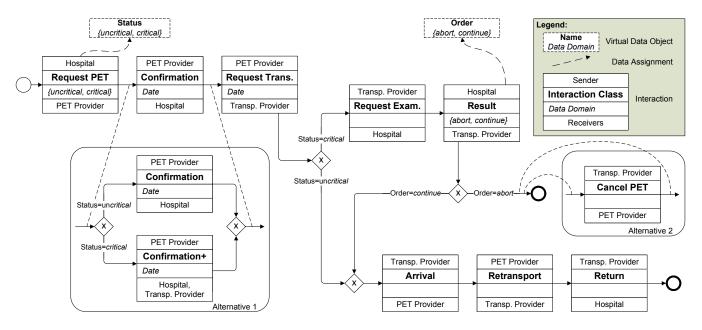


Fig. 2. Patient transportation scenario as DAChor

Alternative 1, which ensures realizability by also sending the confirmation to Transp. Provider, if the patient is in a critical condition. Obviously, implicit knowledge of Transp. Provider about the value of virtual data object Status is sufficient to ensure realizability. This makes the definition of proper correctness criteria for data-aware interaction models Section III very challenging.

Before defining correctness criteria for DAChors, their behavior has to be formalized. In [9], Decker et al. define the behavior of iBPMN choreographies based on their transformation to Interaction Petri Nets (IP Nets). However, IP Nets are unaware of data. This raises the challenge to first enrich IP Nets as well as their behavior with data, i.e., to design Data-Aware Interaction Nets (DAI Nets). Following this, an appropriate transformation is presented.

The main contribution of this technical report is to introduce a formal framework for data-aware interaction models putting emphasis on correctness. Especially, this framework comprises specific correctness criteria for interaction models (e.g. realizability, clear termination). Note, that the latter exceed traditional correctness and soundness criteria that are known from workflows and interconnection models [5], [13], [14]. Further contributions include the introduction of DAChors and DAI Nets as well as the transformation from DAChors to DAI Nets with well defined behavior.

III. FORMAL FRAMEWORK

This section introduces our formal framework for ensuring correctness of data-aware interaction models. First, the scope of an interaction model is described as *interaction domain* and in terms of *messages* (cf. Def. 1 and 2 in Section III-A). Second, *Data-Aware Choreographies* (DAChors) are introduced as formal meta-model for data-aware interaction modeling (cf. Def. 3 in Section III-B). In Section III-D, the semantics of DAChors is described based on their transformation to *Data-Aware Interaction Nets* (DAI Nets). DAI Nets combine *Interaction Petri Nets* (IP Nets) [9] and *Workflow Nets with Data (WFD Nets)* [12] (cf. Def. 5 in Section III-C). Their behavior is described in terms of *markings* and *execution traces* (cf. Def. 8–10 in Section III-E). Def. 12 introduces *conversations* representing the observable parts of an execution trace (i.e., exchanged messages). Finally, *partner views* are defined (cf. Def. 14). Based on traces, conversations, and views, we then introduce *correctness criteria* for DAI Nets and DAChors respectively (cf. Def. 11, 13, and 15). Fig. 3 provides an overview of the main elements of our formal framework and their relations.

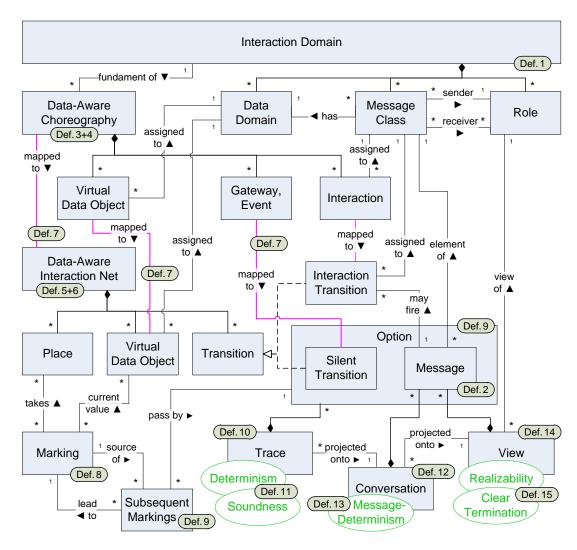


Fig. 3. Overview of our formal framework

A. Interaction Domains and Messages

This section defines the basic elements of data-aware interaction modeling in terms of an *interaction domain*. The latter contains *roles* to differentiate the partners as well as *message classes* and related *data domains*. Furthermore, the notion of *message* (cf. Def. 1 and 2 and Example 1).

Definition 1 (Interaction Domain).

An interaction domain is a tuple $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$, with

- \mathcal{R} is a set of roles,
- \mathcal{D} is a set of data domains; each $D \in \mathcal{D}$ represents a finite set of values,
- C is a set of message classes,
- $dom_C: C \to \mathcal{D}$ is a function assigning to each message class a data domain,
- $s_C: C \to \mathcal{R}$ assigns the sender to each message class,
- $r_C: C \to 2^{\mathbb{R}}$ assigns the set of receivers to each message class,
- ϵ is the empty value.

Further, we define $\Omega_{\mathcal{I}} := \{\epsilon\} \cup \bigcup_{D \in \mathcal{D}} D$ as the set of all values.

Based on Def. 1, Def. 2 introduces the notion of *message*. A message constitutes an instance of a message class. Furthermore, we introduce several sets of messages.

Definition 2 (Messages).

Let $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$ be an interaction domain. Then: A message in \mathcal{I} is a tuple $\mu = (c, x) \in C \times \Omega_{\mathcal{I}}$, with

- $c \in C$ is the corresponding message class, and
- $x \in dom_C(c)$ is the message content transferred.

Furthermore, we define:

- $\Sigma_c := \{(c', x) \in C \times \Omega_{\mathcal{I}} \mid c' = c \land x \in dom_C(c')\}$ as set of all messages corresponding to message class $c \in C$,
- $\Sigma_{\mathcal{I}} := \bigcup_{c \in C} \Sigma_c$ as set of all messages corresponding to interaction domain \mathcal{I} ,
- $\Sigma_{R\to} := \{(c,v) \in \Sigma_{\mathcal{I}} | s_C(c) = R\}$ as set of all messages sent by role $R \in \mathcal{R}$,
- $\Sigma_{\rightarrow R} := \{(c, v) \in \Sigma_{\mathcal{I}} | R \in r_C(c)\}$ as set of all messages received by role R,
- $\Sigma_R := \Sigma_{R \to} \cup \Sigma_{\to R}$ as set of all messages corresponding to role R, i.e. sent or received by R

Example 1 (Basic Notions).

Consider the interaction model of the patient transportation scenario from Fig. 2. Its interaction domain is $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$ with:

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\mathcal{R} = \{ \text{Hospital}, \text{PET Provider}, \text{Transp. Provider} \}
\mathcal{D} = \{D_{\epsilon} = \{\epsilon\}, D_{Status} = \{uncritical, critical\}, D_{Order} = \{abort, continue\}, D_{Date} = \{1.1.1900, \dots, 31.12.2099\}
C = \{Request\ PET, Confirmation, Request\ Trans., Request\ Exam., Result, Arrival, Retransport, \}
       Return, Confirmation+, Cancel PET}
s_C(Request PET)
                          = Hospital
                                                   r_C(Request PET)
                                                                             = {PET provider}
s_C(Confirmation)
                          = PET provider
                                                   r_C(Confirmation)
                                                                             = {Hospital}
                                                   r_C(Request\ Trans.) = \{Transp.\ Provider\}
s_C(Request\ Trans.) = PET provider
s_C(Request Exam.)
                         = Transp. Provider r_C(Request Exam.)
                                                                            = {Hospital}
                                                                             = {Transp. Provider}
s_C(Result)
                          = Hospital
                                                   r_C(Result)
s_C(Arrival)
                       = Transp. Provi
= PET provider
                         = Transp. Provider
                                                   r_C(Arrival)
                                                                             = {PET provider}
s_C(Retransport)
                                                   r_C(Retransport)
                                                                            = {Transp. Provider}
s_C(Return)
                          = Transp. Provider
                                                   r_C(Return)
                                                                             = {Hospital}
s_C(Confirmation+) = PET provider
                                                   r_C(Confirmation+)
                                                                             = {Hospital,
                                                                                 Transp. Provider
s_C(Cancel\ PET)
                          = Transp. Provider
                                                   r_C(Cancel\ PET)
                                                                             = {PET provider}
dom_C(Request PET)
                          = D_{Status}
                                                   dom_C(Confirmation)
                                                                             = D_{Date}
dom_C(Request\ Trans.) = D_{Date}
                                                   dom_C(Request\ Exam.) = D_{\epsilon}
dom_C(Result)
                          = D_{Order}
                                                   dom_C(Arrival)
                                                                             = D_{\epsilon}
dom_C(Retransport)
                                                   dom_C(Return)
                          = D_{\epsilon}
                                                                             = D_{\epsilon}
                                                   dom_C({\it Cancel PET})
dom_C(Confirmation+) = D_{Date}
\Sigma_{\mathcal{I}} = \{ (Request\ PET, uncritical), (Request\ PET, critical), (Result, abort), (Result, continue), \}
        (Request\ Exam., \epsilon), (Arrival, \epsilon), (Confirmation, 1.1.1900), \dots, (Confirmation, 31.12.2099),
        (Confirmation+, 1.1.1900), \dots, (Confirmation+, 31.12.2099), (Retransport, \epsilon), (Return, \epsilon),
        (Request\ Trans., 1.1.1900), \dots, (Request\ Trans., 31.12.2099), (Cancel\ PET, \epsilon)
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Based on the interaction domain from Def. 1, we define the notion of *data-aware choreography* (DAChor). DAChor enriches BPMN choreography models with virtual data objects, a data assignment relation, and guards.

Definition 3 (Data-Aware Choreography; DAChor).

Let $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$ be an interaction domain. Then: A Data-Aware Choreography (DAChor) over \mathcal{I} is a tuple $DAC = (N, I, G, e_s, E_e, G_+^s, G_+^m, G_{d\times}^s, G_{e\times}^s, G_{\times}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$, with

- N is the set of nodes being the disjoint conjunction of the set of interactions I and the set of gateways and events G. In turn, the latter is the disjoint conjunction of the start event $\{e_s\}$, the set of events E_e , the set of AND-splits G_{t}^s , the set of AND-mergers G_{t}^m , the set of data-based XOR-splits G_{t}^s , and the set of XOR-mergers G_{t}^m ,
- V is the set of virtual data objects,
- $class: I \rightarrow C$ assigns a message class to each interaction,
- $\rightarrow \subseteq (N E_e) \times (N \{e_s\})$ is the interaction flow relation,
- $\rightarrow \subseteq I \times V$ is the data assignment relation,
- $dom_V: V \to \mathcal{D}$ is a function assigning a domain to each virtual data object,
- $grd: (\rightarrow) \rightarrow \mathcal{G}_V$ is a function assigning a guard to each interaction flow.

The set of guards \mathcal{G}_V is defined as the set of propositional logic formulas over propositions of the form v = s or the form $v \in \{s_1, s_2, \ldots, s_n\}$. Thereby, $v \in V$ is a *virtual data object* and $s, s_1, s_2, \ldots, s_n \in dom_V(v)$ are values of the related data domain. If a guard $g \in \mathcal{G}_V$ uses a virtual data object $v \in V$, we denote this as $v \stackrel{\epsilon}{\sim} g$. Note that a guard can be constantly *true*. In this case, we omit it in the graphical representation of the DAChor (cf. Fig 2). In the following, we introduce the *well-formedness* of DAChors. Example 2 then provides a formal description of our scenario from Fig. 2.

Definition 4 (Well-Formed DAChor).

A DAChor is well-formed, iff the following properties hold:

- the start event, each interaction, and each merge node have exactly one successor, i.e., $\forall n \in \{e_s\} \cup I \cup G_+^m \cup G_\times^m : |\{n' \in N | n \rightarrow n'\}| = 1$
- each split node has at least one successor, i.e., $\forall g^s \in G^s_+ \cup G^s_{d\times} \cup G^s_{e\times} : |\{n \in N | g^s \to n\}| \ge 1$
- each end event, each interaction, and each split node have exactly one predecessor, i.e., $\forall n \in E_e \cup I \cup G^s_+ \cup G^s_{dx} \cup G^s_{ex} : |\{n' \in N | n' \rightarrow n\}| = 1$
- each merge node has at least one predecessor, i.e., $\forall g^m \in G^m_+ \cup G^m_\times : |\{n \in N | n \to g^m\}| \geq 1$
- each event-based XOR-split is solely followed by interactions, i.e., $\forall g_{e\times}^s \in G_{e\times}^s : \{n \in N | g_{e\times}^s \to n\} \subseteq I$
- guards of interaction flows are constantly true unless the source of an interaction flow is a data-based XOR-split, i.e., $grd((n_1, n_2)) \neq true \Leftrightarrow n_1 \in G^s_{d\times}$
- the data of an interaction is solely assigned to variables of the same data domain, i.e., $\forall i \in I, \forall v \in V : i \rightarrow v \Rightarrow dom_C(class(i)) = dom_V(v)$.
- there is no cycle solely consisting of gateways, i.e., $\nexists g_0, g_1, \dots g_n \in G : g_0 \rightarrow g_1 \rightarrow \dots \rightarrow g_n \rightarrow g_0.$

Example 2 (DAChor).

Consider the scenario from Fig. 2. Basing its interaction domain \mathcal{I} (cf. Example 1) we can describe the given scenario as DAChor DAC = $(N, I, G, e_s, E_e, G_+^s, G_+^m, G_{d\times}^s, G_{e\times}^s, G_{\times}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$:

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I = \{i_1, \dots, i_8\}, E_e = \{e_e^1, e_e^2\}, V = \{Status, Order\}
G_{d\times}^s = \{g_{d\times}^{s1}, g_{d\times}^{s2}\}, G_{\times}^m = \{g_{\times}^m\}, G_{+}^s = G_{+}^m = G_{e\times}^s = \emptyset,
\rightarrow = \{(i_1, Status), (i_5, Order)\}
\rightarrow = \{(e_s, i_1), (i_1, i_2), (i_2, i_3), (i_3, g_{d\times}^{s1}), (g_{d\times}^{s1}, i_4), (g_{d\times}^{s1}, g_{\times}^{m}), (i_4, i_5), (i_5, g_{d\times}^{s2}), (g_{d\times}^{s2}, e_e^1), (g_{d\times}^{s2}, g_{\times}^{m}), (g_{\times}^{m}, i_6), (g_{\times}^{m}, i_{\times}^{m}), (g_{\times}^{m}, i_{
                        (i_6, i_7), (i_7, i_8), (i_8, e_e^2)
     class(i_1) = Request PET
                                                                                                                                                                                        class(i_2) = Confirmation | class(i_3) | = Request Trans.
     class(i_4) = Reguest Exam. |class(i_5)| = Result
                                                                                                                                                                                                                                                                                                                                                                        class(i_6) = Arrival
     class(i_7) = Retransport |class(i_8)| = Return
    dom_V(Status) = D_{status}
                                                                                                                                                                                                                         dom_V(Order) = D_{order}
                                                                                                    = Status = critical \mid grd\left((g_{d\times}^{s1}, g_{\times}^{m})\right) = Status = uncritical
     grd\left(\left(g_{d\times}^{s1},i_{4}\right)\right)
     grd\left(\left(g_{d\times}^{s2},e_{e}^{1}\right)\right)
                                                                                                   = Order = abort
                                                                                                                                                                                                                  grd((g_{d\times}^{s2}, g_{\times}^{m})) = Order = continue
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C. Data-Aware Interaction Net

We introduce the notion of *Data-Aware Interaction Net* (DAI Net). It combines IP Nets [9] and WFD Nets [12]: Hence, the main elements of a DAI Net are places and transitions. To add data, these elements are enriched with variables and guards on transitions as known from WFD Nets. Furthermore, DAI Nets allow assigning message classes to transitions. Like in IP Nets, respective transitions are denoted as *interaction transitions*. Finally, all other transitions are called *silent transitions*.

Definition 5 (Data-Aware Interaction Net; DAI Net).

Let $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$ be an interaction domain. Then, a Data-Aware Interaction Net (DAI Net) over \mathcal{I} is a tuple $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$, where

- P is the set of places; P can be partitioned into the initial place p_{in} , the set of ordinary places P_o , and the set of final places P_{fi} ,
- T is the set of transitions; T can be partitioned into the sets of silent transitions T_S and the set of interaction transitions T_I ,
- V is the set of variables,
- $class: T_I \rightarrow C$ is a function assigning a message class to each interaction transition,
- $\rightarrow \subseteq ((P P_{fi}) \times T) \cup (T \times (P \{p_{in}\}))$ is the flow relation,
- $\rightarrow \subseteq T_I \times V$ is the data assignment relation. It expresses that the data of an interaction transition is assigned to the related variable,
- $dom_V: V \to \mathcal{D}$ is a function assigning a data domain to each variable,
- $grd: T \to \mathcal{G}_V$ is a function assigning a guard to each interaction flow relation.

Further, we define

- $\Sigma_{\#} := \bigcup_{i \in T_I} \Sigma_{class(i)}$ as the set of all messages corresponding to #
- $P^{\to t} := \{ p \in P | p \to t \}$ as the set of all places preceding t
- $P^{\leftarrow t} := \{p \in P | t \rightarrow p\}$ as the set of all places succeeding t
- $P^{\leftrightarrow t} := \{ p \in P | p \rightarrow t \land t \rightarrow p \}$ as the set of the faraway places of t

As below Def. 3, the set of guards \mathcal{G}_V is defined the set of propositional logic formulas over propositions of the form v = s or the form $v \in \{s_1, s_2, \ldots, s_n\}$. Thereby, $v \in V$ is a *variable* and $s, s_1, s_2, \ldots, s_n \in dom_V(v)$ are values of the related data domain. If a guard $g \in \mathcal{G}_V$ uses a variable $v \in V$, we denote this as $v \in \mathcal{G}_V$. Note that a guard can be constantly *true*. In this case, we omit it in the graphical representation of the DAI Net (cf. Fig 4). In the following, we introduce the *well-formedness* of DAI Nets. Then, we introduce a mapping from DAChor to DAI Nets and show that this mapping preserves the property of well-formedness.

Definition 6 (Well-Formed DAI Net).

A DAI Net is well-formed, iff the following properties hold:

- each transition has at least one preceding and one succeeding place, i.e., $\forall t \in T : \exists p_1, p_2 \in P : p_1 \rightarrow t \rightarrow p_2$
- the content of an interaction transition is solely assigned to variables of the same data domain, i.e., $\forall t_i \in T_I, \forall v \in V : t_i \rightarrow v \Rightarrow dom_C(class(t_i)) = dom_V(v)$.
- there exists no cycle solely consisting of places and silent transitions, i.e., $\nexists p_0, p_1, \dots p_n \in P, t_0, t_1, \dots t_n \in T_S : p_0 \to t_0 \to p_1 \to t_1 \to \dots \to p_n \to t_n \to p_0.$

D. Mapping DAChors to DAI Nets

In Section III-C, we introduced DAI Nets to define the behavior of DAChors. Based on this we can now define a mapping from data-aware choreographies to DAI Nets. This mapping is based on the approach proposed by Decker et al. [9] who define the behavior of iBPMN Choreographies through their transformation to IP Nets.

```
Definition 7 (Mapping DAChors to DAI Nets).
Let DAC = (N, I, G, e_s, E_e, G^s_+, G^m_+, G^s_{dx}, G^s_{ex}, G^m_\times, V, class, \rightarrow, \rightarrow, dom_V, grd) be a DAChor (cf. Def. 3). Then,
DAC can be mapped to a DAI Net defined as \# := (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd'), with
 P
                     := \{p_{(n_1,n_2)} | (n_1,n_2) \in \to \land n_1 \notin G_{ex}^s \}
                                                                                                                                           interaction flow
                     := p_{(e_s,n)} \in P, whereby e_s \rightarrow n \in N
                                                                                                                                               start event
 p_{in}
                     := \{p_{(n,e_e)}|p_{(n,e_e)} \in P \land e_e \in E_e\} \subseteq P
                                                                                                                                                end events
 P_{fi}
 P_o
                     := P - (\{p_{in}\} \cup P_{fi})
                     := \{t_{g_+} | g_+ \in G_+^s \cup G_+^m \}
 T_{+}
                                                                                                                                            AND gateways
                     \begin{array}{ll} := & \{t^s_{(g^s_\times,n)} | g^s_\times \in G^s_d \times \wedge n \in N \wedge g^s_\times \to n\} \\ := & \{t^m_{(n,g^m_\times)} | g^m_\times \in G^m_\times \wedge n \in N \wedge n \to g^m_\times\} \end{array} 
 T_{\mathsf{x}}^{s}
                                                                                                                              data-based XOR-split gateways
 T_{\times}^{m}
                                                                                                                                      XOR-merge gateways
                     := \{t_i | i \in I\}, \quad T_S := T_+ \cup T_\times^s \cup T_\times^m, \quad T := T_S \cup T_I \text{ interactions } \}
 T_I
 class'(t_i)
                    := class(i)
                                                                                                                                   message class assignment
                     := \ \{ (p_{(n_1,n_2)},t_{n_2}) | n_1 \to n_2 \ \land \ n_1 \notin G^s_{e^\times} \ \land \ n_2 \in I \cup G^s_+ \cup G^m_+ \}
                                                                                                                                interactions/AND-gateways in
                     \cup \quad \{(t_{n_1}, p_{(n_1, n_2)}) | n_1 \to n_2 \land n_1 \in I \cup G_+^s \cup G_+^m \}
                                                                                                                               interactions/AND-gateways out
                     \cup \{(p_{(n_1,n_2)}, t_{(n_1,n_2)}^m) | n_1 \to n_2 \land n_2 \in G_{\times}^m \}
                                                                                                                                            XOR-merge in
                     \cup \{(t_{(n_0,n_1)}^m, p_{(n_1,n_2)}) | n_0 \to n_1 \to n_2 \land n_1 \in G_{\times}^m \}
                                                                                                                                            XOR-merge out
                     \cup \{(p_{(n_1,n_2)},t^s_{(n_2,n_3)})|n_1 \to n_2 \to n_3 \land n_2 \in G^s_{d\times}\}
                                                                                                                                    data-based XOR-split in
                     \cup \quad \{(t^s_{(n_1,n_2)},p_{(n_1,n_2)})|n_1 \to n_2 \land n_1 \in G^s_{dx}\}
                                                                                                                                   data-based XOR-split out
                     \cup \{(p_{(n_0,n_1)},t_{n_2})|n_0 \to n_1 \to n_2 \land n_1 \in G_{e_{\times}}^s\}
                                                                                                                                      event-based XOR-split
                     := \{(t_i, v) | (i, v) \in \rightarrow \}
                                                                                                                                    data assignment relation
                            \begin{cases} grd((g_{\times}^{s}, n)), & iff \ t = t_{(g_{\times}^{s}, n)} \in T_{\times}^{s} \\ true, & else \end{cases}
 qrd'(t)
                                                                                                                                          guard assignment
```

Theorem 1 states that the mapping from DAChors to DAI Nets preserves well-formedness. The application to our example is shown in Example 3 and Fig. 4.

Theorem 1 (Preservation of Well-Formedness).

Let DAC be a DAChor that is mapped to a DAI Net #. If DAC is well-formed, # is well-formed as well.

We now prove Theorem 1. Our proof consists of three parts that correspond to the three properties of well-formedness for DAI Nets. First, we prove the first property, i.e., each transition has at least one preceding and one succeeding place:

Proof 1 (Preservation of Well-Formedness (Property 1)).

Let $DAC = (N, I, G, e_s, E_e, G_+^s, G_+^m, G_{d\times}^s, G_{e\times}^s, G_{\times}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a well-formed DAChor. DAC is mapped to the DAI Net #. The latter is defined as $\# := (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$, whereby

$$\begin{aligned} & \text{Case } 1 \colon t = t_{g_{+}} \in T_{+} = \{t_{g_{+}} | g_{+} \in G_{+}^{s} \cup G_{+}^{m} \} \\ & \Rightarrow \exists n_{1}, n_{2} \in N : n_{1} \rightarrow g_{+} \rightarrow n_{2} \\ & \Rightarrow \exists p_{(n_{1},g_{+})}, p_{(g_{+},n_{2})} \in P : (p_{(n_{1},g_{+})}, t_{g_{+}}), (t_{g_{+}}, p_{(g_{+},n_{2})}) \in \rightarrow' \\ & \text{i.e., } p_{(n_{1},g_{+})} \rightarrow' t \rightarrow' p_{(g_{+},n_{2})}. \end{aligned}$$

$$\begin{aligned} & \text{Case } 2 \colon t = t_{(g_{x}^{s},n_{1})}^{s} \in T_{x}^{s} \coloneqq \{t_{(g_{x}^{s},n_{1})}^{s} | g_{x}^{s} \in G_{d}^{s} \times \wedge n_{1} \in N \wedge g_{x}^{s} \rightarrow n_{1} \} \\ & \Rightarrow \exists n_{2}, n_{1} \in N : n_{2} \rightarrow g_{x}^{s} \rightarrow n_{1} \\ & \Rightarrow \exists p_{(n_{2},g_{x}^{s})}, p_{(g_{x}^{s},n_{1})} \in P : (p_{(n_{2},g_{x}^{s})}, t_{(g_{x}^{s},n_{1})}^{s}), (t_{(g_{x}^{s},n_{1})}^{s}, p_{(g_{x}^{s},n_{1})}) \in \rightarrow' \\ & \text{i.e., } p_{(n_{2},g_{x}^{s})} \rightarrow' t \rightarrow' p_{(g_{x}^{s},n_{1})} \end{aligned}$$

$$\begin{aligned} & \text{Case } 3 \colon t = t_{(n_{1},g_{x}^{m})}^{m} \in T_{x}^{m} \coloneqq \{t_{(n_{1},g_{x}^{m})}^{m} | g_{x}^{m} \in G_{x}^{m} \wedge n_{1} \in N \wedge n_{1} \rightarrow g_{x}^{m} \} \\ & \Rightarrow \exists n_{1}, n_{2} \in N : n_{2} \rightarrow g_{x}^{m} \rightarrow n_{1} \end{aligned}$$

$$\Rightarrow \exists p_{(n_{1},g_{x}^{m})}, p_{(g_{x}^{m},n_{2})} \in P : (p_{(n_{1},g_{x}^{m})}, t_{(n_{1},g_{x}^{m})}^{m}), (t_{(n_{1},g_{x}^{m})}^{m}, p_{(g_{x}^{m},n_{2})}) \in \rightarrow' \end{aligned}$$

$$\text{i.e., } p_{(n_{1},g_{x}^{m})} \rightarrow' t \rightarrow' p_{(g_{x}^{m},n_{2})} \end{aligned}$$

$$\begin{aligned} & \text{Case } 4 \colon t = t_{i} \in T_{I} \coloneqq \{t_{i} | i \in I \} \\ & \Rightarrow \exists n_{1}, n_{2} \in N : n_{1} \rightarrow i \rightarrow n_{2} \end{aligned}$$

$$\text{Subcase } 4.1 \colon n_{1} \in G_{e_{x}}^{s} \\ & \Rightarrow \exists p_{(n_{0},n_{1})}, p_{(i,n_{2})} \colon (p_{(n_{0},n_{1})}, t_{i}), (t_{i}, p_{(i,n_{2})}) \in \rightarrow' \end{aligned}$$

$$\text{i.e., } p_{(n_{0},n_{1})} \rightarrow' t \rightarrow' p_{(i,n_{2})} \end{aligned}$$

$$\text{Subcase } 4.2 \colon n_{1} \notin G_{e_{x}}^{s}$$

$$\Rightarrow \exists p_{(n_{1},i)}, p_{(i,n_{2})} \colon (p_{(n_{1},i)}, i), (i, p_{(i,n_{2})}) \in \rightarrow' \end{aligned}$$

$$\text{i.e., } p_{(n_{1},i)} \rightarrow' t \rightarrow' p_{(i,n_{2})} \end{aligned}$$

Thus, a transition has at least one preceding and one succeeding place, consequently the first property holds. □

Second, we prove that the data of an interaction transition is solely assigned to variables of the same data domain:

Proof 2 (Preservation of Well-Formedness (Property 2)).

Let $DAC = (N, I, G, e_s, E_e, G_+^s, G_+^m, G_{d\times}^s, G_{e\times}^s, G_{\times}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a well-formed DAChor. DAC is mapped to a DAI Net #. The latter is defined as # := $(P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$. Let $t_i \in T_I$ be an interaction transition and $v \in V$ a virtual data object of #. Then:

$$(t_i, v) \in \rightarrow' \Rightarrow (i, v) \in \rightarrow$$

$$\Rightarrow dom_C(class(t_i)) = dom_C(class(i)) = dom_V(v)$$

Thus, data assignments are correct, consequently the second property holds. \Box

Finally, we prove the third property, i.e., there exists no cycle solely consisting of places and silent transitions. For this purpose, we define a function gate, which assigns to each silent transition in # a gateway in DAC. Furthermore, we show that the gates of two silent transitions are connected if the silent transitions are connected by a place in Lemma 1. trans

Definition (Gate of a Silent Transition).

Let $DAC = (N, I, G, e_s, E_e, G^s_+, G^m_+, G^s_{dx}, G^s_{ex}, G^m_\times, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a well-formed DAChor. DACis mapped to a DAI Net $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$. Then:

The function $gate: T_S \to (G - G_{ex}^s): t_s \mapsto gate(t_s)$ assigns to each silent transition its gate, with

$$gate(t_s) := \begin{cases} g_+, & \text{iff } t = t_{g_+} \in T_+ = \{t_{g_+} | g_+ \in G_+^s \cup G_+^m \} \\ g_\times^s, & \text{iff } t = t_{(g_\times^s, n_1)} \in T_\times^s := \{t_{(g_\times^s, n_1)}^s | g_\times^s \in G_d^s \times \wedge n_1 \in N \wedge g_\times^s \to n_1 \} \\ g_\times^m, & \text{iff } t = t_{(n_1, g_\times^m)}^m \in T_\times^m := \{t_{(n_1, g_\times^m)}^m | g_\times^m \in G_\times^m \wedge n_1 \in N \wedge n_1 \to g_\times^m \} \end{cases}$$

Lemma 1 (Connected Silent Transitions imply Connected Gateways).

Let $DAC = (N, I, G, e_s, E_e, G_+^s, G_+^m, G_{dx}^s, G_{ex}^s, G_{x}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a well-formed DAChor. DACis mapped to a DAI Net $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd').$

If there are two silent transitions t_s^1 , $t_s^2 \in T_S$ that are connected by a place $p_{(n_1,n_2)} \in P$ (cf. Def.7), i.e., $t_s^1 \to p \to t_s^2$ Then holds $gate(t_s^1) \rightarrow gate(t_s^2)$ holds.

Proof 3 (Connected Silent Transitions imply Connected Gateways (Lemma 1)).

Let $DAC = (N, I, G, e_s, E_e, G^s_+, G^m_+, G^s_{dx}, G^s_{ex}, G^m_\times, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a (well-formed) DAChor that is mapped to a DAI Net #, The latter is defined as $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$:

$$\begin{array}{l} \textit{Case 1: } t_{s}^{1} = t_{g_{+}} \in T_{+} \ and \ g_{+} \coloneqq gate(t_{s}^{1}) = \in G_{+}^{s} \cup G^{m} \\ \textit{Subcase 1.1: } t_{s}^{2} = t_{g_{+}}' \in T_{+} \ and \ g_{+}' \coloneqq gate(t_{s}^{2}) \in G_{+}^{s} \cup G^{m} \\ \Rightarrow (t_{g_{+}}, p_{(a,b)}), (p_{(a,b)}, t_{g_{+}'}) \in \rightarrow' \Rightarrow \ a = g_{+} \land b = g_{+}' \Rightarrow g_{+} \rightarrow g_{+}' \\ \textit{Subcase 1.2: } t_{s}^{2} = t_{(g_{s}^{s}', n_{2})}' \in T_{s}^{s} \ and \ g_{s}^{s}' \coloneqq gate(t_{s}^{2}) \in G_{d}^{s} \times \\ \Rightarrow (t_{g_{+}}, p_{(a,b)}), (p_{(a,b)}, t_{(g_{s}^{s}', n_{2})}') \in \rightarrow' \Rightarrow \ a = g_{+} \land b = g_{s}^{s'} \Rightarrow g_{+} \rightarrow g_{s}^{s'} \\ \textit{Subcase 1.2: } t_{s}^{2} = t_{(n_{2}, g_{s}^{m}')}' \in T_{s}^{m} \ and \ g_{s}^{m'} \coloneqq gate(t_{s}^{2}) \in G_{s}^{m} \\ \Rightarrow (t_{g_{+}}, p_{(a,b)}), (p_{(a,b)}, t_{(n_{2}, g_{s}^{m'})}') \in \rightarrow' \Rightarrow \ a = g_{+} = n_{2} \land b = g_{s}^{m'} \Rightarrow g_{+} \rightarrow g_{s}^{m'} \\ \textit{Case 2: } t_{s}^{1} = t_{(g_{s}^{s}, n_{1})}^{s} \in T_{s}^{s} \ and \ g_{s}^{s} \coloneqq gate(t_{s}^{2}) \in G_{d}^{s} \times \\ \textit{Subcase 2.1: } t_{s}^{2} = t_{(g_{s}^{s}', n_{2})}' \in T_{s}^{s} \ and \ g_{s}^{s'} \coloneqq gate(t_{s}^{2}) \in G_{s}^{s} \cup G^{m} \\ \Rightarrow (t_{(g_{s}^{s}, n_{1})}^{s}, p_{(a,b)}), (p_{(a,b)}, t_{g_{+}'}) \in \rightarrow' \Rightarrow \ a = g_{s}^{s} \land b = n_{1} = g_{+}' \Rightarrow g_{s}^{s} \rightarrow g_{+}' \\ \textit{Subcase 2.2: } t_{s}^{2} = t_{(g_{s}^{s}', n_{2})}' \in T_{s}^{s} \ and \ g_{s}^{s'} \coloneqq gate(t_{s}^{2}) \in G_{d}^{s} \times \\ \Rightarrow (t_{(g_{s}^{s}, n_{1})}, p_{(a,b)}), (p_{(a,b)}, t_{(g_{s}^{s}', n_{2})}') \in \rightarrow' \Rightarrow \ a = g_{s}^{s} \land b = n_{1} = g_{+}' \Rightarrow g_{s}^{s} \rightarrow g_{s}^{s'} \\ \textit{Subcase 2.3: } t_{s}^{2} = t_{(n_{2}, g_{s}^{m}')}' \in T_{s}^{m} \ and \ g_{s}^{m'} \coloneqq gate(t_{s}^{2}) \in G_{d}^{m} \\ \Rightarrow (t_{(g_{s}^{s}, n_{1})}, p_{(a,b)}), (p_{(a,b)}, t_{(n_{2}, g_{s}^{m}')}') \in \rightarrow' \Rightarrow \ a = g_{s}^{s} \land b = n_{1} = g_{s}^{s'} \Rightarrow g_{s}^{s} \rightarrow g_{s}^{s'} \\ \textit{Subcase 3.1: } t_{s}^{2} = t_{g}^{m} \land b = n_{1} = g_{s}^{m'} \Rightarrow g_{s}^{m} \land b = g_{s}^{m} \land b = g_{s}^{m'} \Rightarrow g_{s}^{m} \land b = g_{s}^{m} \land b = g_{s}^{m'} \Rightarrow g_{s}^{m} \land b = g_{s}^{m'} \land b = g_{s}^{m'} \Rightarrow g_{s}^{m} \land b = g_{s}^{m'} \land b =$$

$$\Rightarrow (t^{m}_{(n_{1},g^{m}_{\times})},p_{(a,b)}),(p_{(a,b)},t^{s}_{(g^{s'}_{\times},n_{2})}') \in \to' \Rightarrow a = g^{m}_{\times} \land b = g^{s'}_{\times} \Rightarrow g^{m}_{\times} \to g^{s'}_{\times}$$

$$Subcase \ 3.2: \ t^{2}_{s} = t^{m}_{(n_{2},g^{m'}_{\times})}' \in T^{m}_{\times} \ and \ g^{m'}_{\times} := gate(t^{2}_{s}) \in G^{m}_{\times}$$

$$\Rightarrow (t^{m}_{(n_{1},g^{m}_{\times})},p_{(a,b)}),(p_{(a,b)},t^{m}_{(n_{2},g^{m'}_{\times})}') \in \to' \Rightarrow a = g^{m}_{\times} = n_{2} \land b = g^{m'}_{\times} \Rightarrow g^{m}_{\times} \to g^{m'}_{\times}$$

Thus, for all cases $gate(t_s^1) \rightarrow gate(t_s^2)$ holds and Lemma 1 is proven. \Box

We use function gate and Lemma 1 to prove the third property of well-formedness of DAI Nets by contradiction:

Proof 4 (Preservation of Well-Formedness (Property 3)).

Assume property 3 of Theorem 1 is violated. Then: There exists a well-formed $DAC = (N, I, G, e_s, E_e, G_+^s, G_+^m, G_{d\times}^s, G_{e\times}^s, G_{\times}^m, V, class, \rightarrow, \rightarrow, dom_V, grd)$ that is mapped to a non-well-formed DAI Net #, with # = $(P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$:

- $\Rightarrow \exists p_0, p_1, \dots p_n \in P, t_0, t_1, \dots t_n \in T_S : p_0 \to' t_0 \to' p_1 \to' t_1 \to' \dots \to' p_n \to' t_n \to' p_0$
- $\Rightarrow gate(t_0) \rightarrow gate(t_1) \rightarrow \cdots \rightarrow gate(t_n) \rightarrow gate(t_0)$
- $\Rightarrow \exists g_0 \coloneqq gate(t_0), g_1 \coloneqq gate(t_1), \dots g_n \coloneqq gate(t_n) : g_0 \to g_1 \to \dots \to g_n \to g_0$

This contradicts our assumption. Thus, the third property holds. □

According to Proofs 1-4, the mapping from DAChor to DAI Net (cf. Def. def:mapping) preserves all three properties of well-formedness. Thus, Theorem 1 holds.

```
Example 3 (Transformation).
```

The DAChor DAC = $(N, I, G, e_s, E_e, G^s_+, G^m_+, G^s_{d\times}, G^s_{e\times}, G^m_{\times}, V, class, \rightarrow, \rightarrow, dom_V, grd)$ from Example 2 is mapped to the DAI Net $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class', \rightarrow', \rightarrow', dom_V, grd')$ as follows (cf. Fig. 4):

E. Behavior of DAI Nets

Since DAI Nets are based on both WFD Nets and IP Nets, we use token semantics (i.e., tokens assigned to places and token changes) to define their behavior. Together with the values of variables, tokens define the marking

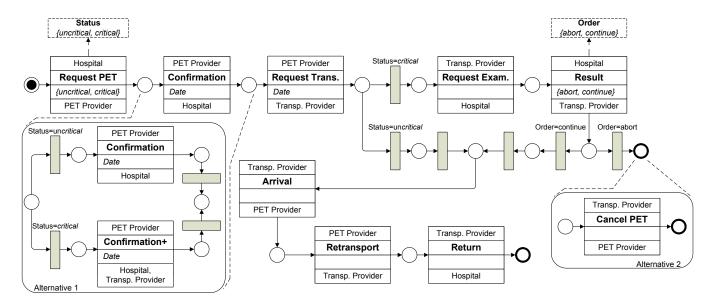


Fig. 4. DAI Net derived for the patient transportation scenario

of a DAI Net. Each Interaction Net starts with an initial marking, with exactly one token placed in the initial place p_{in} and each variable having the empty value ϵ . A marking is called *final*, if all tokens belong to final places of P_{fi} . A transition t is activated under marking m, iff all directly preceding places of t contain at least one token, and the guard of t is evaluable and evaluates to true.

Definition 8 (DAI Net Markings and Activated Transitions).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net. Then: A marking of # is a tuple m = (0, val) with

- $\odot: P \to \mathbb{N}_0$ assigns to each place the number of corresponding tokens,
- $val: V \to \Omega_{\mathcal{I}}$ assigns to each variable its current value; val(v) is either the empty value ϵ or an element of the variable's domain, i.e., $val(v) \in dom_V(v) \cup \{\epsilon\}$.

Additionally, for each DAI Net # we define the

- set of all markings $\mathcal{M}_{\#}$, whereby $\mathcal{M}_{\#} := \{ m = (\odot, val) \mid m \text{ is a marking of } \# \}$
- initial marking $m_{\#}^{in} := (\odot_{in}, val_{in}) \in \mathcal{M}_{\#}$, whereby

- set of all final markings $\mathcal{F}_{\#}$, whereby $\mathcal{F}_{\#} := \{(\odot, val) \in \mathcal{M}_{\#} | \forall p \in P : \odot(p) \neq 0 \Leftrightarrow p \in P_{fi}\}$
- transition activation relation $\sim \subseteq \mathcal{M}_{\#} \times T$. $m \sim t$ denotes that marking $m \in \mathcal{M}_{\#}$ activates transition $t \in T$, iff the following conditions hold:
 - 1) $\forall p \in P^{\to t} : \odot(p) \ge 1$,
 - 2) $\forall v \stackrel{\epsilon}{\sim} qrd(t) : val(v) \neq \epsilon$,
 - 3) grd(t) is satisfied for marking m

If a transition is activated, it may fire and lead from the current marking to a subsequent one. More precisely, one token is taken from each preceding place and one is added to each succeeding place. Silent transitions fire immediately when they become activated. Activated interaction transitions fire, if and only if a message of the corresponding message class is sent. In this case, the value of the message is assigned to virtual data objects as

expressed by the data assignment relation. Note that a message can only be sent if an interaction transition of the related message class is activated and no silent transition is activated (cf. Def. 9).

Definition 9 (Options and Subsequent Markings of DAI Nets).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net, an $m = (\odot, val), m' = (\odot', val') \in \mathcal{M}_{\#}$ be two related markings. Then:

- $\mathcal{O}_{\#} := T_S \cup \Sigma_{\#}$ is the set of all options on #.
- $opt_{\#}: \mathcal{M}_{\#} \to 2^{\mathcal{O}_{\#}}: m \mapsto \{o \in \mathcal{O}_{\#} | \exists m' \land m \xrightarrow{o} m'\}$ maps each marking m to the options available under m.
- $m \stackrel{o}{\to} m'$ expresses that m leads to m' by applying option $o \in opt_{\#}(m)$ with:

Case 1: $o = t_s \in T_S$ is a silent transition. Then: $m \stackrel{t_s}{\to} m'$ holds, iff each of the following conditions is met:

- 1) $m \rightsquigarrow t_s$,
- 2) $\forall p \in P^{\to t_s} : \odot'(p) = \odot(p) 1$,
- 3) $\forall p \in P^{\leftarrow t_s} : \odot'(p) = \odot(p) + 1$,
- 4) $\forall p \in P^{\leftrightarrow t_s} : \odot'(p) = \odot(p)$,
- 5) $\forall v \in V : val'(v) = val(v)$.

Case 2: $o = \mu = (c, x) \in \Sigma_{\#}$ is a message. Then: $m \xrightarrow{\mu} m'$ holds, iff the following conditions are met:

- 1) $\forall t_s \in T_S : m \not\rightarrow t_s$,
- 2) $\exists t_i \in T_I : m \leadsto t_i \land \mu \in \Sigma_{class(t_i)}$,
- 3) $\forall p \in P^{\to t_i} : \odot'(p) = \odot(p) 1$,
- 4) $\forall p \in P^{\leftarrow t_i} : \emptyset'(p) = \emptyset(p) + 1$,
- 5) $\forall p \in P^{\leftrightarrow t_i} : \emptyset'(p) = \emptyset(p),$
- 6) $\forall v \in V \text{ with } t_i \rightarrow v : val'(v) = x$,
- 7) $\forall v \in V \text{ with } t_i \rightarrow v : val'(v) = val(v).$

Based on Def. 9, the following two theorems can be derived.

Theorem 2 (Separation of Options). Let # be a DAI Net. Then: For each marking, the set of options either contains solely silent transitions or messages or it is empty, i.e.,

$$\forall m \in \mathcal{M}_{\#} : opt_{\#}(m) \neq \emptyset \quad \Rightarrow \quad opt_{\#}(m) \subseteq T_S \oplus opt_{\#}(m) \subseteq \Sigma_{\#}$$

Theorem 3 (Termination of final markings). Let # be a DAI Net. Then: For each final marking, the set of options is empty, i.e.,

$$\forall m \in \mathcal{F}_{\#} : opt_{\#}(m) = \emptyset$$

We prove Theorem 2 and Theorem 3 by contradiction:

Proof 5 (Separation of Options).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net. Then: Assume, Theorem 2 is violated for #:

- $\Rightarrow \exists m \in \mathcal{M}_{\#} : opt_{\#}(m) \neq \emptyset \text{ and } \exists t_0 \in T_S, \mu \in \Sigma_{\#} : t_0, \mu \in opt_{\#}(m)$
- $\Rightarrow \exists m' \in \mathcal{M}_{\#} : m \xrightarrow{\mu} m' \Rightarrow \forall t_s \in T_S : m \not \Rightarrow t_s \Rightarrow m \not \Rightarrow t_0$
- $\Rightarrow t_0 \notin opt_{\#}(m)$

This contradicts our assumption. Thus, Theorem 2 is proven. \Box

Proof 6 (Termination of final markings).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net. Then: Assume, Theorem 3 is violated for #:

```
\Rightarrow \exists m = (\odot, val) \in \mathcal{F}_{\#} : opt_{\#}(m) \neq \emptyset
\Rightarrow \exists t \in T : m \leadsto t \Rightarrow \exists p \in P : \odot(p) \ge 1 \land p \to t
\Rightarrow p \notin P_{fi} \Rightarrow m \notin \mathcal{F}_{\#}
```

This contradicts our assumption. Thus, Theorem 3 is proven. □

Based on Def. 9, we define *traces* on DAI Nets as sequences of options. To be more precise, a trace corresponds to a *related sequence of markings* that starts with the initial marking. If this related sequence of markings ends with a final marking, we denote the trace as *completed*.

Definition 10 (Traces, Prefixes, and Extensions).

(A) Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net and $\tau = (\tau_k)_{k \in [1..n]} \in \mathcal{O}_\#^*$ be a finite sequence of options (i.e. silent transitions and messages) with length $|\tau| =: n \in \mathbb{N}$. Let further $m = (m_k)_{k \in [1..n+1]} \in \mathcal{M}_\#^*$ be a finite sequence of markings with length n + 1. Then:

- $\tau \sim m$ denotes that τ and m are related sequences, iff $\forall l \in [1..n] : m_l \stackrel{\tau_l}{\to} m_{l+1}$ and $m_1 = m_\#^{in}$.
- $last: \mathcal{M}_{\#}^{\star} \to \mathcal{M}_{\#}$ with $(m_k)_{k \in [1..n]} \mapsto m_n$ is a function mapping a sequence of markings to its last marking.
- $\tau \in \mathcal{O}_{\#}^{\star}$ is a trace, iff $\exists m \in \mathcal{M}_{\#}^{\star}$ and $\tau \sim m$. If $last(m) \in \mathcal{F}_{\#}$, we denote τ as completed trace.
- $T_{\#}$ denotes the set of all traces on #.
- $\mathcal{T}^c_{\#}$ denotes the set of all completed traces on #.
- **(B)** Let $L \subseteq M^*$ be a set of finite sequences over a set M and let $a = (a_k)_{k \in [1..n]}, b = (b_k)_{k \in [1..l]}, c = (c_m)_{m \in [1..m]} \in L$ be elements of L, i.e. sequences over M. Then:
 - $a \le b$ (a < b) denotes a is prefix (real prefix) of b and b an extension (real extension) of a, iff $n \le l$ (n < l) and $\forall i \in [1..n] : a_i = b_i$,
 - a + c = b denotes that a is extended by c to b, iff m + n = l, and a is prefix of b, and $\forall i \in [1..m] : c_i = b_{n+i}$,
 - $L^{\unlhd b} := \{a \in L | a \unlhd b\} \ (L^{\unlhd b} := \{a \in L | a \unlhd b\}) \ denotes \ the \ subset \ of \ L \ that \ contains \ all \ prefixes \ (real \ prefixes) \ of \ b \in L, \ and$
 - $L^{\trianglerighteq b} := \{a \in L | b \trianglelefteq a\} \ (L^{\trianglerighteq b} := \{a \in L | b \lhd a\}) \ denotes \ the \ subset \ of \ L \ that \ contains \ all \ extensions \ (real \ extensions) \ of \ b \in L.$

We described the behavior of a DAI Net by means of its traces. We can also use traces to characterize the desired behavioral properties of DAI Nets. The first one is *determinism*. It expresses that a trace is unique in terms of its related markings, i.e., replaying a trace will always lead to the same marking. The second fundamental property is *soundness* in terms of boundedness as well as the absence of deadlocks and lifelocks [15].

Definition 11 (Determinism and Soundness).

(A) We call a DAI Net # deterministic, iff for each trace τ on # there exists exactly one related sequence of markings, i.e., $\forall \tau \in \mathcal{T}_{\#} : |\{m \in \mathcal{M}_{\#}^{\star} | m \sim \tau\}| = 1$.

Let # be a deterministic DAI Net. Then:

 $mark_{\#}$ maps each trace on # to its current marking, i.e. the last marking of the related sequence of markings: $mark_{\#}: \mathcal{T}_{\#} \to \mathcal{M}_{\#}: \tau \mapsto mark_{\#}(\tau) \coloneqq last(m)$, whereby m is defined by $\tau \sim m \in \mathcal{M}_{\#}^*$.

Since # is deterministic, the definition of m is unique. Thus, $mark_{\#}$ is well defined.

- **(B)** We call a deterministic DAI Net # sound, iff the following conditions hold:

 - There exist completed traces on #, i.e., T^c_# ≠ Ø,
 Each trace on # is a prefix of a completed trace, i.e., ∀υ ∈ T_#∃τ ∈ T^c_#: υ ⊴ τ.
 - The set of reachable markings is finite, i.e., $|\{m \in \mathcal{M}_{\#}|\exists \tau \in \mathcal{T}_{\#}: last(\tau) = m\}| \in \mathbb{N}$

Note that the observable behavior of any DAI Net is solely explained through the messages exchanged. Hence, we must abstract from the silent elements of traces (i.e. silent transitions) and define the observable behavior as a conversation being the projection of a trace to its messages (i.e., the part of the trace defining its semantic). In the following, we first introduce projections of sequences.

Definition 12 (Projections and Conversations).

Let A, B be two sets with $B \subseteq A$, and $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a DAI Net and $\tau \in \mathcal{T}_{\#}$ be a trace on #. Then:

- $\Pi_B: A^* \to B^*: a \mapsto \Pi_B(a)$ is the projection function that restricts a sequence $a \in A^*$ to its elements of B,
- $\eta \in \Sigma_{\#}^{\star}$ denotes a conversation on #, iff it is the projection of a trace on # to its messages, i.e., $\exists (\tau) \in \mathcal{T}_{\#} : \Pi_{\Sigma_{\#}}(\tau) = \eta$. η denotes a completed conversation on #, iff it is the projection of a completed
- $C_{\#}$ denotes the set of all conversations on #,
- $\mathcal{C}^c_{\#}$ denotes the set of all completed conversations on #,
- $con_{\#}: \mathcal{T}_{\#} \to \mathcal{C}_{\#}: \tau \mapsto con_{\#}(\tau) := \Pi_{\Sigma_{\#}}(\tau)$ maps each trace to the related conversation.

Example 4 (Traces and Conversations).

Consider the DAI Net # from Example 3. Its set of completed traces $\mathcal{T}^c_{\#}$ consists of traces τ_1 , τ_2 , and τ_3 . Projecting them to their messages leads to the conversations η_1 , η_2 , and η_3 , which build $\mathcal{C}^c_{\#}$:

```
\tau_1 = \langle (\text{Request PET}, uncritical), (Confirmation, 1), (\text{Request Trans.}, 1), t_{(a^{s_1}, a^m)}^s, t_{(a^{s_1}, a^m)}^m, t_{(a
                                                                            (Arrival, \epsilon), (Retransport, \epsilon), (Return, \epsilon) >
                               = < (\textit{Request PET}, critical), (\textit{Confirmation}, \_^1), (\textit{Request Trans.}, \_^1), t^s_{(g^{s_1}_{s_2}, i_4)}, (\textit{Request Exam.}, \epsilon),
                                                                            (Result, abort), t_{(g_{dx}^{s2}, e_e^1)}^s >
   \tau_3 = <(\textit{Request PET}, critical), (\textit{Confirmation}, \_^1), (\textit{Request Trans.}, \_^1), t^s_{(g^{s1}_{dx}, i_4)}, (\textit{Request Exam.}, \epsilon), t^s_{(g^{s1}_{dx}, i_4)}, t^s_{(g^{s1}
                                                                            (\textit{Result}, continue), t^s_{(q^{s2}_{sz}, q^m_y)}, t^m_{(q^{s2}_{sz}, q^m_y)}, (\textit{Arrival}, \epsilon), (\textit{Retransport}, \epsilon), (\textit{Return}, \epsilon) > 
                          \eta_1 = con_\#(\tau_1) := \Pi_{\Sigma_\#}(\tau_1) = \langle (Request\ PET, uncritical), (Confirmation, _1^1), (Request\ Trans., _1^1), (Reques
                                                                                                                                                                                                                                                                                                                               (Arrival, \epsilon), (Retransport, \epsilon), (Return, \epsilon) >
                        \eta_2 = con_\#(\tau_2) \coloneqq \Pi_{\Sigma_\#}(\tau_2) = < (\textit{Request PET}, critical), (\textit{Confirmation}, \_^1), (\textit{Request Trans.}, \_^1),
                                                                                                                                                                                                                                                                                                                                  (Request Exam., \epsilon), (Result, abort) >
                        \eta_3 = con_\#(\tau_3) \coloneqq \Pi_{\Sigma_\#}(\tau_3) \quad = \quad <(\textit{Request PET}, critical), (\textit{Confirmation}, \_^1), (\textit{Request Trans.}, \_^1), (\textit{
                                                                                                                                                                                                                                                                                                                                      (Request Exam., \epsilon), (Result, continue), (Arrival, \epsilon), (Retransport, \epsilon),
                                                                                                                                                                                                                                                                                                                                      (Return, \epsilon) >
```

As aforementioned, the behavior of silent transitions is not observable. Hence, to ensure compatible behavior of participating roles, silent transitions must behave deterministically. In other words, it must be possible to determine the behavior of a DAI Net solely based on the messages exchanged, i.e., message-determinism. First, this requires,

that firing of silent transitions always terminates, i.e., it is impossible to solely execute silent transitions infinitely (cf. Theorem 4). Second, when silent transitions terminate, the set of activated options may only depend on the messages exchanged before, i.e., it should be independent from the order in which the silent transitions were fired.

Theorem 4 (Termination of silent subtraces). On a well-formed DAI Net #, any trace cannot infinitely be continued by silent transitions, i.e.

$$\forall \tau \in \mathcal{T}_{\#}: \exists N \in \mathbb{N} \text{ such that } \forall v \in \mathcal{T}_{\#}^{\trianglerighteq \tau} \text{ with } |\tau| + N < |v| \implies con_{\#}(\tau) \neq con_{\#}(v).$$

To proof Theorem 4, we introduce silent ways, which solely consist of places and silent transitions. With the use of those, we define a ranking function that decreases each time a silent transition is fired.

Definition (Silent Ways).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net. Then:

- A silent way ω from a place $p \in P$ to a silent transition $t \in T_S$ is a sequence of alternating places and silent transitions $\omega = \langle p_0, t_0, p_1, t_1, \ldots, p_n, t_n \rangle \in (PT_S)^*$ with $p = p_0$ and $t = t_n$, whereby holds $p_0 \to t_0 \to p_1 \to t_1 \to \cdots \to p_n \to t_n$,
- $W_{\#}$ as the set of all silent ways on #, and
- $\mathcal{W}_{\#}^{p \to t} := \{ \langle p_0, \dots, t_n \rangle \in \mathcal{W}_{\#} | p_0 = p \land t_n = t \}$ as the set of all silent ways on # from $p \in P$ to $t \in T_S$.

Consider that Def. 6 prohibits cycles of silent transitions. Thus, each place and each transition can occur at least once in a silent way. Consequently, $W_{\#}$ is finite. Obviously, the same applies to each $W_{\#}^{p \to t} \subseteq W_{\#}$

Lemma 2.

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net and $t, t' \in T_S$ be silent transitions. Then:

$$\forall p \in P^{\to t}: \sum_{q \in P^{\to t}} |\mathcal{W}_{\#}^{q \twoheadrightarrow t'}| \ge |\mathcal{W}_{\#}^{p \twoheadrightarrow t'}| \ge \sum_{q \in P^{\leftarrow t}} |\mathcal{W}_{\#}^{q \twoheadrightarrow t'}|$$

Proof 7.

The right inequality $|\mathcal{W}_{\#}^{p \to t'}| \ge \sum_{q \in P^{\leftarrow t}} |\mathcal{W}_{\#}^{q \to t'}|$ holds because of:

$$\forall q \in P^{\leftarrow t} : p \to t \to q \implies \forall q \in P^{\leftarrow t} : \forall \omega = \langle q, t_0, p_1, t_1, \dots p_n, t' \rangle \in \mathcal{W}_{\#}^{q \to t'}$$

$$\Rightarrow \exists ! \ \omega' = \langle p, t, q, t_0, p_1, t_1, \dots p_n, t' \rangle \in \mathcal{W}_{\#}^{p \to t'}$$

The left inequality $\forall p \in P^{\to t} : \sum_{q \in P^{\to t}} |\mathcal{W}_{\#}^{q \to t'}| \ge |\mathcal{W}_{\#}^{p \to t'}|$ holds because of:

$$\begin{split} p \in P^{\to t} & \Rightarrow & \bigcup_{q \in P^{\to t}} \mathcal{W}_{\#}^{q \to t'} \supseteq \mathcal{W}_{\#}^{p \to t'} \\ & \Rightarrow & \sum_{q \in P^{\to t}} |\mathcal{W}_{\#}^{q \to t'}| \ge |\bigcup_{q \in P^{\to t}} \mathcal{W}_{\#}^{q \to t'}| \ge |\mathcal{W}_{\#}^{p \to t'}| \end{split}$$

Thus, Lemma 2 is proven. □

¹For reasons of simplification, we abstract from irrelevant message content in Example 4

Based on Lemma 2 we prove Theorem 4. For this purpose, first, we introduce a transition ranking function ζ that maps a marking and a silent transition to a natural number. Second, based on ζ , we define a net ranking function ξ that bases on maps each marking of a # to a natural number. Finally, we show, that ξ is decreased each time a silent transition fires (cf. Lemma 3). Thus, ξ is an upper bound to the number of steps (i.e., firings of silent transitions) that can be done in # until the net terminates.

Definition (Ranking Functions). Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net. Then, the transition ranking function ζ is defined as below:

- $\zeta_{\#}: \mathcal{M}_{\#} \times T_S \to \mathbb{N}: ((\odot, val), t) \mapsto \zeta((\odot, val), t)$ is the transition ranking function that maps a marking and a silent transition to a natural number, with $\zeta((\odot, val), t) \coloneqq \sum_{p \in P} \odot(p) * |\mathcal{W}_{\#}^{p \to t}|$.
- $\xi: \mathcal{M}_{\#} \to \mathbb{N}: m \mapsto \xi(m)$ is the net ranking function that maps a marking to a natural number, with $\xi(m) \coloneqq \sum_{t \in T_S} \zeta(m, t)$.

Lemma 3 (The Rank of a Net Decreases when a Silent Transition Fires).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net. Further, $m, m' \in \mathcal{M}_\#$ be two markings of #, with $m = (\odot, val), m' = (\odot', val')$. Finally, $t_0 \in T_S$ be a silent transition of # with $m \stackrel{t_0}{\rightarrow} m'$. Then:

$$\xi(m) > \xi(m')$$

To prove Lemma 3, we partition ξ based on its definition for each $t_0 \in T_S$ that may be fired:

$$\xi(m) = \sum_{t \in T_S} \zeta(m, t) = \zeta(m, t_0) + \sum_{t \in T_S - \{t_0\}} \zeta(m, t)$$

We consider both parts (i.e. $\zeta(m, t_0)$ and $\sum_{t \in T_S - \{t_0\}} \zeta(m, t)$) on its own to show that ξ decreases, whenever a silent transition t_0 is fired.

Lemma 4 (The Rank of a Fired Silent Transition Decreases).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net. Further, $m, m' \in \mathcal{M}_\#$ be two markings of #, with $m = (\odot, val), m' = (\odot', val')$. Finally, $t_0 \in T_S$ be a silent transition of # with $m \xrightarrow{t_0} m'$. Then:

$$\zeta(m,t_0) > \zeta(m',t_0)$$

Proof 8 (The Rank of a Fired Silent Transition Decreases (Lemma 4)).

$$\zeta(m, t_{0}) = \sum_{p \in P} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) = \sum_{p \in P \to t} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \\
= \sum_{p \in P \to t} \left(\left(\odot'(p) + 1 \right) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot'(p) * 0 \right) + \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \\
= \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(|\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \\
> \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P \to t} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \\
= \sum_{p \in P} \left(\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) = \zeta(m', t_{0})$$

Thus, $\zeta(m,t_0) > \zeta(m',t_0)$ holds and Lemma 4 is proven. \Box

Lemma 5 (The Rank-Sum of all Unfired Silent Transitions does not Increase).

Let $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a deterministic and sound DAI Net. Further, $m, m' \in \mathcal{M}_\#$ be two markings of #, with $m = (\odot, val), m' = (\odot', val')$. Finally, $t_0 \in T_S$ be a silent transition of # with $m \xrightarrow{t_0} m'$. Then:

$$\sum_{t \in T_S - \{t_0\}} \zeta(m, t) \ge \sum_{t \in T_S - \{t_0\}} \zeta(m', t)$$

Proof 9 (The Rank-Sum of all Unfired Silent Transitions does not Increase (Lemma 5)).

$$\begin{split} \sum_{t \in T_{S} - \{t_{0}\}} \zeta(m, t) &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P^{\to t}} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P^{\to t}} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P^{\to t}} \left(\odot(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P^{\to t}} \left((\odot'(p) + 1) * |\mathcal{W}_{\#}^{p \to t}| \right) + \sum_{p \in P^{\to t}} \left((\odot'(p) - 1) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) + \sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &\geq \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) + \sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &\geq \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) + \sum_{p \in P^{\to t}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) = \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) = \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left(\sum_{p \in P} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) = \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right) \\ &= \sum_{t \in T_{S} - \{t_{0}\}} \left((\odot'(p) * |\mathcal{W}_{\#}^{p \to t}| \right) \right)$$

Thus,
$$\sum_{t \in T_S - \{t_0\}} \zeta(m, t) \ge \sum_{t \in T_S - \{t_0\}} \zeta(m', t)$$
 holds and Lemma 5 is proven. \square

Having proven that Lemma 4 and Lemma 5 hold, we now can prove Lemma 3:

Proof 10 (The Rank of a Net Decreases when a Silent Transition Fires (Lemma 3)).

$$\xi(m) = \sum_{t \in T_S} \zeta(m, t) = \zeta(m, t_0) + \sum_{t \in T_S - \{t_0\}} \zeta(m, t)$$

$$> \zeta(m', t_0) + \sum_{t \in T_S - \{t_0\}} \zeta(m, t) \ge \zeta(m', t_0) + \sum_{t \in T_S - \{t_0\}} \zeta(m', t) = \sum_{t \in T_S} \zeta(m', t) = \xi(m')$$

Thus, $\xi(m) > \xi(m')$ holds and Lemma 3 is proven. \Box

The termination of silent subtraces (i.e, Theorem 4) is a direct consequence of Lemma 3:

Proof 11 (Termination of Silent Subtraces).

From Lemma 3 results:

$$\forall \tau \in \mathcal{T}_{\#} : \exists N \coloneqq \xi(mark_{\#}(\tau)) \in \mathbb{N} : \forall v \in \mathcal{T}_{\#}^{\trianglerighteq \tau} \text{ with } |\tau| + N < |v| \Rightarrow con_{\#}(\tau) \neq con_{\#}(v).$$

Thus, Theorem 4 is proven. \Box

According to Theorem 4, a DAI Net is *message-deterministic*, if the set of activated messages solely depends on the messages exchanged before (cf. Def. 13).

Definition 13 (Message-Determinism).

We call a deterministic and sound DAI Net # message-deterministic, iff the same sequence of messages always activates the same messages, i.e., the set of activated messages solely depends on the messages exchanged before, i.e.,

$$\forall \tau, \upsilon \in \mathcal{T}_{\#} : \left(opt_{\#}(mark_{\#}(\tau)), opt_{\#}(mark_{\#}(\upsilon)) \subseteq \Sigma_{\#} \wedge \Pi_{\Sigma_{\#}}(\tau) = \Pi_{\Sigma_{\#}}(\upsilon) \right)$$
$$\Rightarrow opt_{\#}(mark_{\#}(\tau)) = opt_{\#}(mark_{\#}(\upsilon))$$

Let # be a deterministic, sound and message-deterministic DAI Net. Then:

 $mo_\#: \mathcal{C}_\# \to 2^{\Sigma_\#}: \eta \mapsto mo_\#(\eta)$ maps each conversation to the set of messages it activates, with $mo_\#(\eta) \coloneqq opt_\#(mark_\#(\tau)), \ \tau \in \mathcal{O}_\#^*$ is defined by $\eta = con_\#(\tau)$ and $opt_\#(mark_\#(\tau)) \subseteq \Sigma_\#$.

Since # is message-deterministic, the definition is unique. Thus, mo# is well defined.

Until now, we solely considered DAI Nets and conversations from a global perspective. However, a role solely knows those messages of a conversation it sends or receives. Thus, in Def. 14 the view of a role on the messages of a conversation is introduced. Further, for each role the set of activated options is defined.

Definition 14 (Views on Conversations and Options).

Let $\mathcal{I} = (\mathcal{R}, \mathcal{D}, C, dom_C, s_C, r_C, \epsilon)$ be an interaction domain and let the tuple $\# = (P, p_{in}, P_o, P_{fi}, T, T_S, T_I, V, class, \rightarrow, \rightarrow, dom_V, grd)$ be a sound, deterministic, and message-deterministic DAI Net. Let further $R \in \mathcal{R}$ be a role. Then we can define the following views

- vc^R_#: C*_# → Σ*_R: (η_k)_{k∈[1..n]} → vc^R_#(η) := Π_{Σ_R}(η) maps each conversation on # to the view of R on it, whereby the view is the projection of the conversation to the messages sent or received by Role R,
 vc^{R→}_#: C*_# → Σ*_R: (η_k)_{k∈[1..n]} → vc^{R→}_#(η) := Π_{Σ_{R→}}(η) maps each conversation on # to the projection of the conversation to the messages sent by Role R,
 vo^R_#: 2^Σ_# → 2^{Σ_R}: M → vo^R_#(M) := M ∩ Σ_R maps each set of messages to its messages that may be sent or

- received by Role R,

 $vo_{\#}^{R\rightarrow}: 2^{\Sigma_{\#}} \rightarrow 2^{\Sigma_{R\rightarrow}}: M \mapsto vo_{\#}^{R\rightarrow}(M) \coloneqq M \cap \Sigma_{R\rightarrow}$ maps each set of messages to its messages that may be sent by Role R.

Based on Def. 14, we can define realizability. It denotes DAI Nets to be deterministic from the viewpoint of a role. Further, clear termination is defined, which indicates that a role can determine when it sent or received its last message.

Definition 15 (Realizability, Clear Termination).

Let # be a deterministic, sound, and message-deterministic DAI Net. Then, for a role $R \in \mathcal{R}$:

• # is realizable, iff the messages role R may send solely depend on the messages R has sent and received before, i.e.,

$$\forall R \in \mathcal{R} : \forall \eta, \kappa \in \mathcal{C}_{\#} : vc_{\#}^{R}(\eta) = vc_{\#}^{R}(\kappa) \Rightarrow vo_{\#}^{R}(mo_{\#}(\eta)) = vo_{\#}^{R}(mo_{\#}(\kappa))$$

• # clearly terminates, iff it solely depends on the messages R has sent and received before whether further interaction with R will occur, i.e.,

$$\forall R \in \mathcal{R} : \forall \eta \in \mathcal{C}_{\#}^{c} \nexists \kappa \in \mathcal{C}_{\#} : vc_{\#}^{R}(\eta) \triangleleft vc_{\#}^{R}(\kappa)$$

An important issue concerns decidability of the introduced properties of DAI Nets and DAChors; i.e., determinism, soundness, message-determinism, realizability, and clear termination (cf. Def. 11-15). Basically, these properties are decidable. Due to lack of space, we omit a discussion in this technical report.

IV. RELATED WORK

In the context of workflows [1], [16] and SOA [17], correctness has been discussed for a long time [15]. The approaches presented [12], [18] consider data as well. The two paradigms for modeling choreographies (i.e. interconnection and interaction models) are compared in [19]. Examples of interconnection models are BPMN 2.0 Collaborations [3] and BPEL4Chor [4]. There are several approaches that discuss the verification classic soundness criteria (i.e. boundedness, absence of deadlocks, absence and lifelocks) of distributed and collaborative workflows and service orchestrations [5], [13], [14], [20]–[24]. Some object-aware [25], [26] and data-driven approaches [27], [28] use data dependencies to interconnect processes and to define process interactions. Examples of interaction models (i.e., the paradigm we apply) include Service Interaction Patterns [7], WSCDL [8], iBPMN Choreographies [9], and BPMN 2.0 Choreographies [3]. Our approach has been mainly inspired by [9], which defines the behavior of iBPMN Choreographies through their transformation to Interaction Petri Nets and further discusses correctness and realizability. Realizability of interaction models is also discussed in [10], [29]. Furthermore, [11] provides a tool for checking realizability of BPMN 2.0 Choreographies. However, all these approaches do not explicitly consider the data exchanged by messages and used for routing decisions.

In [30], [31], state-based conversation protocols are introduced, which are aware of message contents. The messages (and data) exchanged trigger state transitions. Thus, different data may trigger different transitions. However, conversation protocols do not support the modeling of parallelism since they are state-based. Furthermore, realizability of conversation protocols requires that at every state each partner is either able to send or receive a message or to terminate (autonomy condition). This condition strongly restricts parallelism. For example, consider a choreography solely consisting of two parallel branches: In the upper branch partner A sends a message m_1 to partner B and partner B sends message m_2 to A in the lower branch. Obviously, the autonomy condition is violated although the choreography is realizable (cf. Def. 15). Hence, conversation protocols do not constitute interaction models in our point of view. Thus, to our best knowledge the framework presented within this technical report is the first one that considers realizability and clear termination of data-aware interaction models.

V. SUMMARY AND OUTLOOK

Our vision is to provide sophisticated support for distributed and collaborative workflows. To foster this vision, we base our work on the analysis of scenarios from different domains. In essence, we learned that data support is practically relevant for interaction models from a variety of domains.

Further, this technical report introduced a formal framework for data-aware interaction models and described how correctness can be ensured. The main parts of our framework include DAChors and DAI Nets as well as the transformation of DAChors to DAI Nets. Further, the behavior of DAI Nets is defined. Other fundamental contributions are the definitions of correctness criteria for data-aware interaction models. The latter include message-determinism, realizability, and clear termination. In future work, we will extend our framework to support asynchronous message exchange and related correctness properties. Finally, we will develop algorithms for efficiently checking correctness of data-aware interaction models. In this context, we plan to apply abstraction strategies to large data domains similar to [32].

Considering the data perspective is important but may be not sufficient to enable sophisticated support for distributed and collaborative workflows. The time perspective [33]–[35] and the resource perspective [36], [37] should be considered as well in the context of interaction modeling.

However, correctness criteria discussed in this technical report solely address structural and behavioral correctness. As outlined in [38] semantic correctness (i.e., business process compliance) is challenging for distributed and collaborative processes as well. Thus, we will try to transfer the results of our previous work about business process compliance [32], [39]–[41] to distributed and collaborative processes.

Fields of application of our research may be domains with collaborative and heavily interacting processes, e.g., healthcare domain [42] and automotive domain [28], [43].

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